Annual Report

for the period 1 July 1969 to 30 June 1970

Detector Development

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Hg(1-x)Cd(x)Te

Annual Report

for the period 1 July 1969 to 30 June 1970

Summary:

This report describes some recent efforts to study and understand the performance and the noise properties of state-of-the-art $Hg_{1-x}Cd_xTe$ infrared detectors under low background conditions. Particular emphasis has been placed on the study of the low frequency noise properties of these detectors.

In the main body of the report, Dr. W. J. Moore describes the present approach to noise measurements and detector evaluation along with the first results of his experiments. This part gives a description of experiments in which the probability distributions of noise amplitudes from several $Hg_{1-x}Cd_xte$ elements obtained from Texas Instruments, Inc. have been measured. In all cases the distributions are gaussian. When measurements of the distribution are repeated the variance of the distribution is found to change. Such a situation has been termed "noisy-noise" by Brophy [1]. Additional data is required before the statistics of the distribution of variances can be determined for the $Hg_{1-x}Cd_xTe$ elements.

Appendix A consists of a theoretical paper entitled "Phenomenological Approach to Low Frequency Electrical Noise" by S. Teitler and M. F. M. Osborne which is in press in the Journal of Applied Physics. The development invdves two basic physical ideas. Firstly, the excess noise is ascribed to the fluctuation of the local (in time) reference level to which equilibrium and other white (in the low frequency limit) fluctuations are compared. Secondly, it is

assumed that excess low frequency noise involves a nonlinear process. This implies the noise power in different frequency regions are coupled. A further simple assumption requires the integral of the power spectral density over specially chosen intervals to be a constant. This leads to a 1/f spectrum for the power spectral density. Possible generalizations that provide for other spectral laws are also mentioned.

The calendar of events associated with the aquisition of state-of-the-art $Hg_{1-x}Cd_xTe$ detectors is as follows:

Contract in the amount of $39,994 for three state-of-the-art arrays of $Hg_{1-x}Cd_xTe$ detectors awarded to Texas Instruments, Inc. (Contract Number N00173-70-C-0444)	9 Jan 1970
Delivery of first array	Feb 1970
Delivery of second array	March 1970
Delivery of third* array	March 1970
Delivery of fourth array	May 1970

*Due to the slightly substandard performance of the first array, a second array of the same nominal element size was provided by Texas Instruments, Inc. at no cost.

During fiscal 1970 there have been monthly reports on this task beginning with 1 Oct 1969. With this annual report, it is proposed to return to the quarterly reporting procedure prescribed by ARPA guidelines.

During fiscal 1970, the personnel concerned with this task have been Dr. S. Teitler (principal investigator) and Dr. W. J.

Moore (project scientist). During fiscal 1971, Dr. Moore will assume responsibilities as principal investigator and Dr. P. T. Radoff will participate in the project.

Twenty copies of this report are being sent to the Defense Documentation Center, Cameron Station, Alexandria, Va. 22314.

LOW FREQUENCY NOISE PROPERTIES OF STATE-OF-THE-ART $Hg_{1-x}Cd_xTe$ INFRARED DETECTORS

W. J. Moore

I. Preliminary Considerations.

Infrared detectors have long suffered from electrical noise in excess of that produced by the fundamental Johnson-Nyquist and generation recombination mechanisms. This excess noise has a power spectrum which roughly follows a 1/f dependence and it is referred to as "1/f" noise. It reduces the detectivity of the detector, particularly in systems which must operate with slowly varying signals. In addition, excess noise increases the statistical spread in detectivity of elements in arrays with resultant degradation of the overall array performance.

The objectives of this study are to procure evaluated state-of-the-art $Hg_{1-x}Cd_xTe$ detector arrays with peak sensitivity near 10 microns, to perform additional measurements of responsivity and noise under a variety of background conditions, and to compare the statistics of the "1/f" noise in these detectors with theoretical models for "1/f" noise mechanisms.

Recent studies of amplitude probability distributions of "1/f" noise in carbon resistors by Brophy [1] indicate that this noise differs statistically from Johnson-Nyquist noise. Such statistical studies offer the possibility of testing models of "1/f" noise generation mechanisms more precisely than is possible using the traditional power spectrum measurements alone.

Traditional experimental studies of noise in semiconductors have been limited to measurements of power spectra. The usual

system for power spectra measurements consist of a tunable narrow band amplifier followed by an rms reading voltmeter. The voltmeter reading is proportional to the rms amplitude of the Fourier components near the selected frequency. Filters have been used in this way to make measurements at frequencies down to about 10^{-3} hertz [2]. Similar measurements have been made down to 5×10^{-5} hertz using frequency multiplication methods with analog magnetic tape [3] and photographic film [4]. Recently, Mansour, Hawkins and Bloodworth [5] describe a technique of making power spectra by Fourier analyzing digital tapes made using a system which periodically sampled and digitized the waveform of interest. Their concern was in the spectral region below 1 hertz but their work represents the first use of digital computers in the analysis of electrical noise.

Another advance in the analysis of the nature of "1/f" noise is the work of Brophy [1] and Hooge and Hoppenbrouwers [6] who determine the characteristics of the amplitude probability distribution of "1/f" noise from carbon resistors and CdSe films respectively. Brophy finds that the variance of the probability distribution is itself a variable and changes when the experiment is repeated. Hooge and Hoppenbrouwers find that the variance of the distribution of noise from CdSe layers is constant. In both cases the noise power varies approximately as 1/f and the probability distributions are gaussian.

In addition to the above studies of amplitude probability density of "1/f" noise, Brophy [7], has determined the probability distribution for the intervals between zero crossings for both Johnson-Nyquist and "1/f" noise.

All of the above studies with the exception of those of Mansour, Hawkins and Bloodworth require that the experiment be repeated for each type of analysis. It is not convenient to perform more than one analysis on one sample of noise.

A much more general and useful system for noise analysis has been designed in the present work. This system allows all of the measurements described above and in addition allows one to make more complex tests of noise theories than those discussed above. The system consists of an analog tape recorder, facilities for sampling and digitizing data recorded on the analog tape and placing the digitized samples on digital magnetic tape, and a general purpose computer. This system will be described in detail later.

II. Properties of the Purchased $Hg_{1-x}Cd_xTe$ Arrays.

$Hg_{1-x}Cd_xTe$ arrays have been purchased under Contract Number N00173-70-C-0444 with Texas Instruments Inc. This contract called for delivery of three arrays of at least five elements each. Arrays were to be provided with elements having the nominal sizes: 3 x 3 mils, 6 x 6 mils, and 20 x 20 mils. Five elements from each array were to be evaluated at 28°K and 4.2°K with restricted background illumination. Due to slightly substandard performance of the first array delivered (array no. 256 - 17C) a second array of the same nominal element size was provided. The arrays delivered are summarized in Table 1 with comments on their performance.

III. Experimental Details.

Block diagrams of the two systems used in this study are shown in Fig. 1. The system shown in Fig. 1a is essentially the same as

that used previously by Brophy [1] except for the analog tape recorder. Figure 1b shows the more general setup including noise analysis directly utilizing a computer.

In the experimental configuration shown in Fig. 1a, the $Hg_{1-x}Cd_xTe$ detector array is mounted onto a copper block which is attached to a liquid coolant reservoir and is held at a temperature near 78°K or 4.2°K. A cooled opaque cap covers the detector in order to maintain a low level of optical background and a low level of optically produced generation-recombination noise. The detector is biased using a filtered battery supply as shown in Fig. 1c. The resulting current flowing through the sample passes through the primary of a PAR Model Am-1 input transformer (Triad G-4 Geoformer) and produces an amplified fluctuating output in the transformer secondary. This output is further amplified by a PAR Model 113 preamplifier. The transformer is connected for a voltage gain of 51:1. With this system the sample appears to the transformer to be a 120 Ω noise voltage source for noise frequencies greater than about 20 hertz. No distortion due to current in the primary has been observed.

Signals from the amplifier are recorded on analog magnetic tape which is capable of recording frequencies as high as 20 kHz with a signal to noise ratio of 46 db.

The recorded signals are played back at a later date either at the record speed or at a slower speed (in order to get a division of all frequencies present) into an HP 5401B multichannel analyzer used in the sampled voltage analysis mode. When the

waveform has been sampled for a sufficient time (~ 1-2 minutes) the contents of the memory is punched onto paper tape which can be analyzed by computer. When frequency multiplication or division are unnecessary the analog tape recorder is omitted.

When using the generalized system shown in Fig. 1b, the dewar, preamp, and analog recorder are as described above. Signals recorded on the analog tape are played back into a digitizer which can digitize and record more than 10^4 samples per second from the waveform. Frequency division or multiplication can be used for special applications. The unique feature of this system is that the chronological order of samples from the random waveform is preserved.

IV. Results.

Noise from the preamplifier and from biased $Hg_{1-x}Cd_xTe$ detectors held at liquid nitrogen and liquid helium temperatures has been analyzed using the two systems describe above.

Amplitude probability distributions made using the pulse height analyzer confirm the expectation that the noise is normally distributed. An amplitude probability distribution for element B of array Q-351-11C held at about 4.2°K with a bias current of 1.5 ma (slightly above optimum current) is reproduced in Fig. 2.

Skewness and kurtosis are two statistical parameters of a distribution which are independent of gain and so are characteristic of the type of distribution. For a normal distribution they have the values 0 and 3.0 respectively. The skewness and kurtosis given on the graph for the distribution of Fig. 2 were determined using

moments calculated from all 512 experimental points of the distribution and so include data from the wings which would strongly affect the result if the wings deviated from a normal distribution. As indicated on the figure, the resulting values are very close to the theoretical values which suggests that the distribution is gaussian even well out into the wings. We do not yet have sufficient data to determine the statistics of the variances. However, the variances measured do vary, that is, they do not remain constant when the experiment is repeated.

Several noise samples have been digitized using an NRL digitizing facility. Some problem has arisen in attempting to get good resolution from this system due to the lack of a sample and hold circuit ahead of the successive approximation digitizer. Commercial digitizing facilities are available at reasonable rates and will be tested in the near future. Only when it is possible to get digitization with good amplitude resolution will the full capability of the generalized system be realized.

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TABLE 1

Array Number	Nominal Element Size (mils)	True Element Size (mils)	Number of Elements Connected	Comments
Q351-11A	3 x 3	3.5 x 3.0	23	Good array with one particularly good element
256-17C	6 x 6	6.5 x 7.5	23	Average
Q253-27A	6 x 6	6.4 x 6.4	17	Average
Q351-11C	20 x 20	20.5 x 17.9	10	Noisy, not staggered

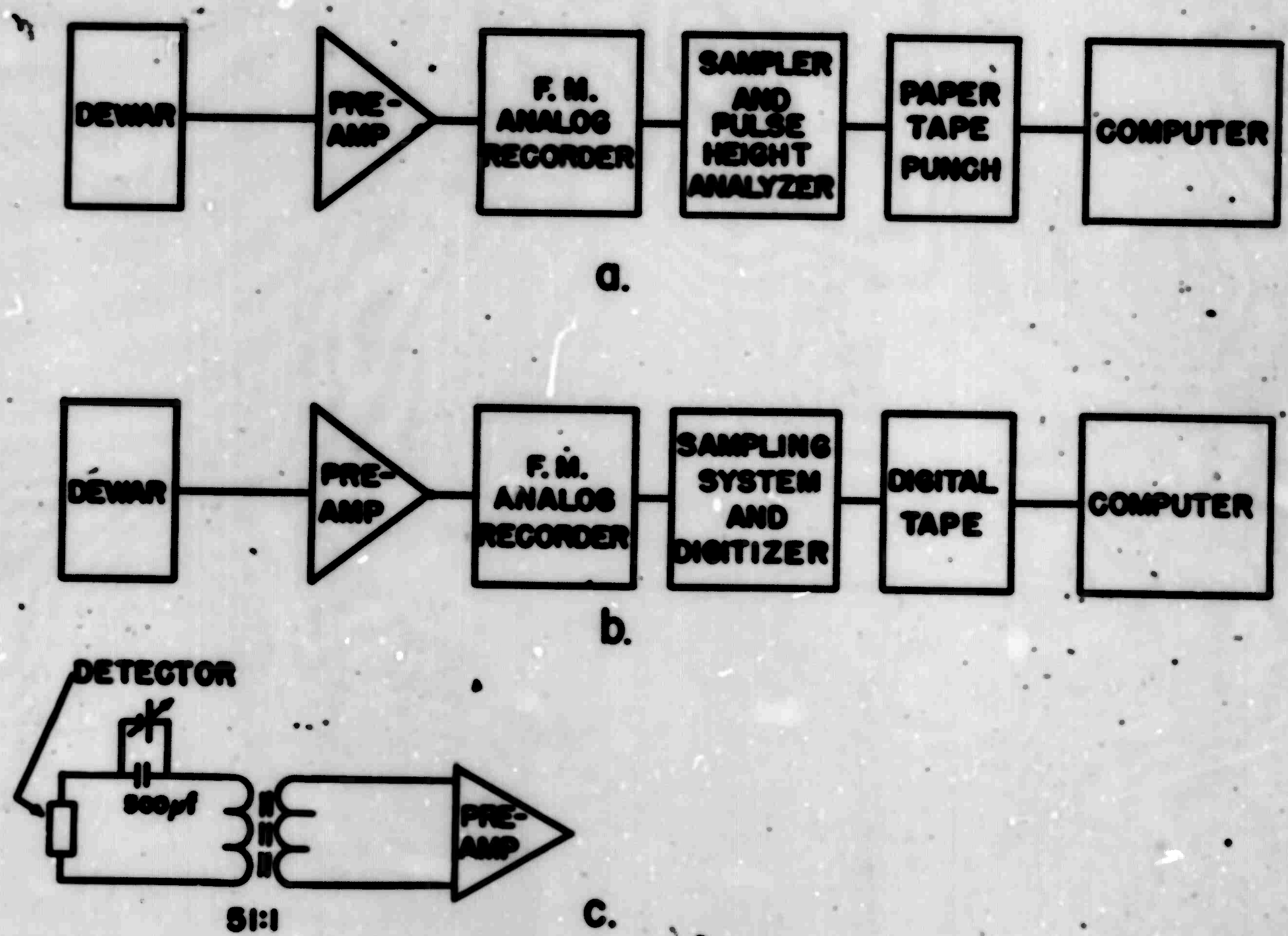
DEWAR
PRE-AMP
F. M. ANALOG RECORDER
SAMPLER AND PULSE HEIGHT ANALYZER
PAPER TAPE PUNCH
COMPUTER
a.
DEWAR
PRE-AMP
F. M. ANALOG RECORDER
SAMPLING SYSTEM AND DIGITIZER
DIGITAL TAPE
COMPUTER
b.
DETECTOR
500pf
51:1
PRE-AMP
c.

Fig. 1

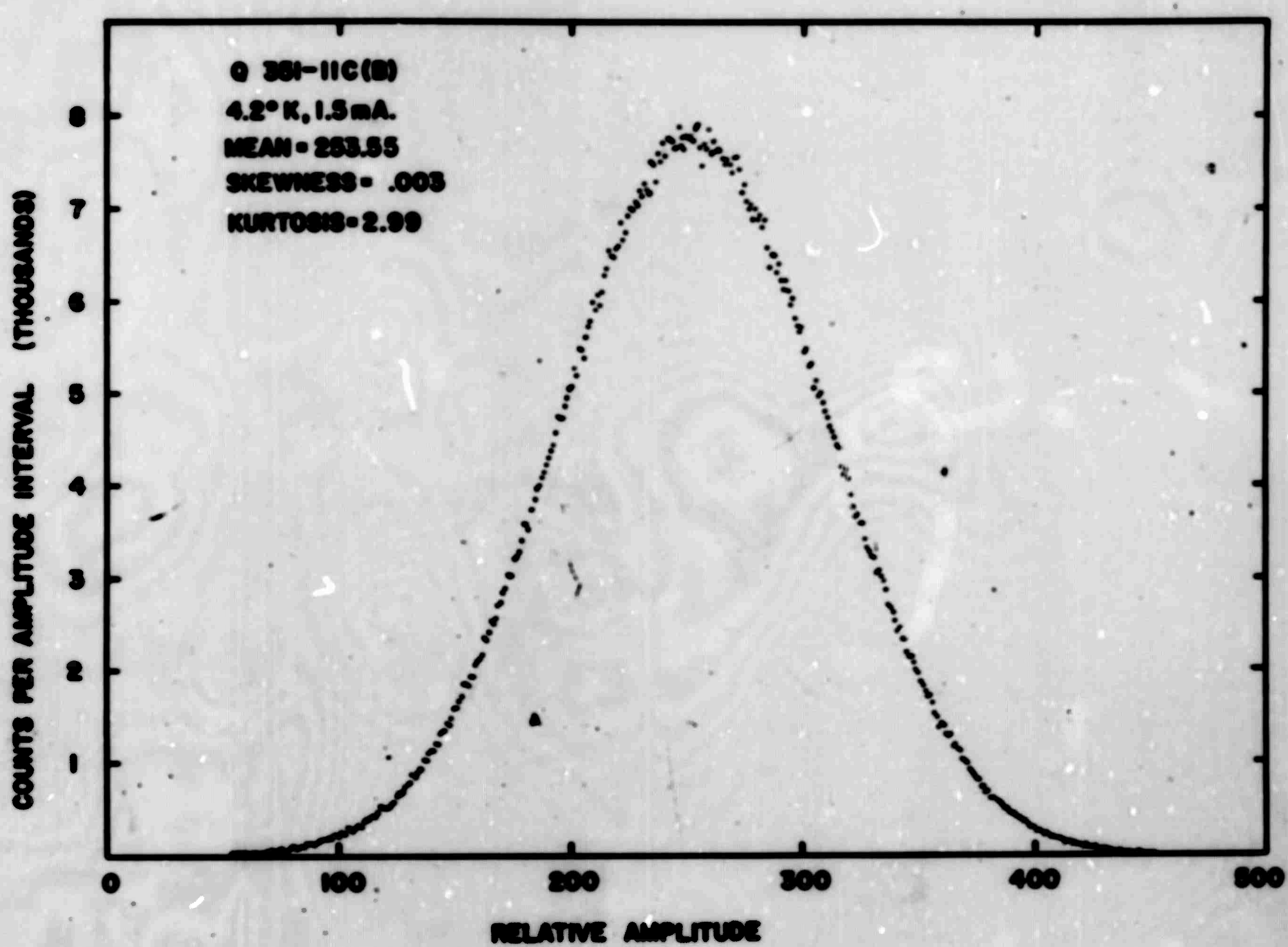
Q 351-11C(B)
4.2° K, 1.5 mA.
MEAN = 253.55
SKEWNESS = .003
KURTOSIS = 2.99
COUNTS PER AMPLITUDE INTERVAL (THOUSANDS)
RELATIVE AMPLITUDE
0
100
200
300
400
500
1
2
3
4
5
6
7
8

Fig. 2

APPENDIX A

Phenomenological Approach to Low Frequency Electrical Noise*†

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Abstract: A phenomenological description of excess noise in a nonequilibrium system is presented. The development involves two basic physical ideas. Firstly, the excess noise is ascribed to the fluctuation of the local (in time) reference level to which equilibrium and other white (in the low frequency limit) fluctuations are compared. Secondly, it is assumed that excess low frequency noise involves a nonlinear process. This implies the noise power in different frequency regions are coupled. A further simple assumption requires the integral of the power spectral density over specially chosen intervals to be a constant. This leads to a 1/f spectrum for the power spectral density. Possible generalizations that provide for other spectral laws are also mentioned.

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†This paper is to appear in the Journal of Applied Physics, 1970.

I. Introduction

Low frequency electrical noise usually has a component which is proportional to the square of the driving current and which has a spectral density varying approximately as the inverse of the frequency. This so-called excess noise is not readily described within the usual mathematical framework employed in the description of fluctuation phenomena. However some specific models have been used in a description of the low frequency spectrum in some specific instances.[1,2,3,4,5,6] Also not long ago, Mandelbrot[7] provided a generalization of the mathematical framework for describing fluctuation phenomena which he applied to the low frequency noise problem.[8] More recently, Mandelbrot and Van Ness[9] have introduced the concept of fractional Brownian motion[10] and related it to low frequency noise.

In the present paper we attempt to describe low frequency electrical noise within a general framework of fluctuation phenomena in nonequilibrium systems. Our approach is deductive in nature and our aim is to broadly label the essential physical processes that may lead to excess low frequency noise.

A nonequilibrium system is subjected to external driving forces. We are interested in the response of the system to the external forces as revealed in variables of flow; e.g. a charge current, a mass current, etc. It is usual to define a fluctuation for such variables at a given time as the difference of the value at that time and its mean value. This meanvalue,

in practice, is defined as the time average of the variable over the empirical record. Now equilibrium and other white (in the low frequency limit) noises are all characterized by a correlation time. Therefore any time interval long compared to these correlation times forms an appropriate averaging interval.

It is physically feasible then to consider two distinct aspects of the fluctuations, namely, (1) fluctuations around a reference level local in time; and (2), fluctuations of the local reference level. The fluctuations about the local reference level are just those that give rise to the white (in the low frequency limit) noise. However, the fluctuations of the local reference level itself represents, from our viewpoint, one of the essentially new ingredients leading to excess noise in nonequilibrium systems. We refer to this fluctuation of the local reference level as the average current deviation.

In addition, as will develop in the construction of our phenomenological model, we invoke an unspecified nonlinear mechanism in order to provide coupling among the spectral components of the average current deviation.

II. The Average Current Deviation

Our goal in this section is to analyze the fluctuations in the local reference level which we describe and denote as the average current deviation. The end result is the power spectral density for the average current deviation.

We proceed in the time domain in a manner similar to that used by von Weizsäcker[11] in the space domain to describe high Reynold's number turbulence phenomena. However it should be emphasized that the present discussion is not directly a turbulence theory. We do not deal with the spatial domain at all. Nevertheless our phenomenological results are consistent with a turbulence mechanism for low frequency noise in semiconductors[5,6] in that such a mechanism provides a nonlinear coupling among the frequency components of the noise.

Consider a very long time record of length $\mathcal{T}$. The average of the variable of interest, e.g. the current, over this record is $\bar{I}$

$$\bar{I} = \mathcal{T}^{-1} \int_{-\mathcal{T}/2}^{\mathcal{T}/2} I\,(t)\,dt \qquad (2.1)$$

At any point on the record, the fluctuation may be represented as the difference between the value of the current at that point, and the average value.

Consider now an interval T_o centered on the point of interest t_a. This interval is chosen to be much smaller than the total record $\mathcal{T}$ but much larger than the correlation times of white (in the low frequency limit) noises. This means that spectral properties for frequencies $f > T_o^{-1}$ may be investigated in this interval and also that the interval T_o is sufficiently large so either it or a small fraction of it, both provide a satisfactory averaging interval for typical processes characterized by a correlation time.

The average of the current over the interval T_o centered on t_a is

$$I^a\,(T_o) = T_o^{-1} \int_{t_a - T_o/2}^{t_a + T_o/2} I(t)\,dt \qquad (2.2)$$

In general the average over the interval T_o will not coincide with the average over the entire empirical interval $\mathcal{T}$.

$$I^A (T_o) - \bar{I} \neq 0 \tag{2.3}$$

We may express the fluctuation at t_a in the following form

$$I (t_a) - \bar{I} = I (t_a) - I^A (T_o) + I^A (T_o) - \bar{I} \tag{2.4}$$

Now T_o was chosen much larger than the correlation times of the white (in the low frequency limit) noises. Consider the smaller time interval $T_1 = T_o/R = r\, T_o$ where R is a small integer.[12] Then we can form the average of the current over the interval T_1 centered on t_a,[13] namely $I^A (T_1)$. The fluctuation at t_a may then be written

$$I (t_a) - \bar{I} = I (t_a) - I^A (T_1) + \underbrace{I^A (T_1) - I^A (T_o)}_{I_1^A} + \underbrace{I^A (T_o) - \bar{I}}_{I_o^A} \tag{2.5}$$

Similarly we can continue to subdivide the orginal interval T_o to form $T_2 = r^2\, T_o$, $T_3 = r^3\, T_o$, --- $T_s = r^s\, T_o$. The subdivision is stopped at T_s which representsthe smallest interval which can serve as an averaging interval for white (in the low frequency limit) noises. The fluctuation at t_a may then be written generalizing Eq. (2.5)

$$I (t_a) - \bar{I} = I (t_a) - I^A (T_s) + \Sigma_{n = o}^{n = s}\, I_n^A \tag{2.6}$$

where

$$I_o^A = I^A (T_o) - \bar{I}$$

$$I_n^A = I^A (T_n) - I^A (T_{n-1}), \quad s \geq n > o \tag{2.7}$$

We identify $I (t_a) - I^A (T_s)$ as the usual fluctuation at t_a, now with respect to a local reference level. Then $\Sigma_{n = o}^{n = s}\, I_n^A$ is the fluctuation at t_a intrinsic to the nonequilibrium situation and we designate it the average current deviation.

It is now important to understand the meaning of the foregoing analysis of the average current deviation in terms of its spectral content. The integral of the current over a particular interval T_n averages out contributions to the average current, which belong to frequencies greater than T_n^{-1} and contains the effects of contributions with frequencies less than T_n^{-1}. The v. Weizsäcker subtraction procedure used in defining the I_n in Eq. (2.7) provides that a given I_n^A only has significant contributions in the frequency range $f_{n-1} = T_{n-1}^{-1}$ to $f_n = T_n^{-1}$.[14] The frequency difference is

$$f_n - f_{n-1} \equiv \Delta f_n = R^n (R-1)/RT_0$$

$$= f_n (R-1)/R \qquad (2.8)$$

One sees then that the size of the frequency interval is an increasing function of n. If one defines the frequency resolution as $f_n/\Delta f_n$, it follows that the frequency resolution is a constant independent of n, albeit not very large.

$$f_n/\Delta f_n = R/(R-1) \qquad (2.9)$$

Alternatively it might seem desirable to consider an analysis with a fixed frequency interval. This can be achieved by considering averages over decreasing intervals of size T_0/ℓ, (ℓ = 1, 2, 3, ---). Then the interval size is constant independent of ℓ and equal to T_0^{-1}. However the frequency resolution $f_\ell/\Delta f_\ell$ is equal to ℓ and therefore increases with increasing frequency. The maximum frequency resolution in the v. Weizsäcker decomposition occurs when R is chosen to be 2. Thus the decomposition with fixed interval size almost always has a frequency resolution greater than that of the v. Weizsäcker decomposition. However if one is interested in the coupling of frequency modes, it seems valuable to consider different frequency ranges with the same resolution. Indeed in the next section we shall see the special role played by the constant frequency resolution interval in our phenomenological model of 1/f noise.

Before that, we finish our analysis of the spectral content of the average current deviation by finding the associated power spectral density. As we have discussed the individual contributions to the average current deviation at t_a, namely the I_n^a, have a spectral nature in that each represents a contribution in a respective spectral interval Δf_n.[12] Thus on a continuous frequency scale we may define a <u>spectral</u> average current deviation at t_a as follows.

$$I_n^a/\Delta f_n \to I^a(f) \text{ [illegible]} \equiv \Delta I^a(f) \tag{2.10}$$

Here the arrow denotes the passage from a discrete description (n) to a continuous variable f.

The empirical ensemble average of $|\Delta I^a(f)|^2$ is obtained by averaging $|\Delta I^a(f)|^2$ over all possible values of t_a which are, by definition, confined to the interval $[-(\mathcal{T} - T_o)/2, \ (\mathcal{T} - T_o)/2]$. We denote this average by $\overline{|\Delta I(f)|^2}$. Then the power spectral density may be written in the form

$$\begin{aligned} P(f) &= 2\,(\mathcal{T} - T_o)^{-1}\,\overline{|\Delta I(f)|^2} \\ &\approx 2\,\mathcal{T}^{-1}\,\overline{|\Delta I(f)|^2} \qquad , f > 0 \end{aligned} \tag{2.11}$$

Here the factor 2 arises since we limit f to positive values and the last approximate equation follows since $T_o \ll \mathcal{T}$.

Finally we note that consistency with the experimental facts about 1/f noise is obtained if we assume that the noise power spectrum is proportional to the power in the driving current. For then, the power spectral density is proportional to the square of the driving current.

III. The Spectral Law

In the previous section we have postulated a possible source of noise in a nonequilibrium electrical system as the average current deviation. We have shown how a power spectral density may be obtained from such a mechanism. In this section we shall use as a starting point only (1) the assumed existence of a power spectral density for low frequency noise without specifying its source. The principal further ingredients of the discussion are (2) an assumption of stationarity of the nonequilibrium process and concurrent total power balance; (3) an assumption of nonlinearity so that the total power balance can be separated into a power balance among intervals in the frequency spectrum in a special way.

The total power in the noise is proportional to the integration of the power spectral density over the total spectrum. Consider the integral of the power spectral density over only part of the spectrum. This represents the component of the noise power in a particular spectral range. The assumption of nonlinearity implies that noise power in different spectral regions are interconnected. Indeed we consider a dynamical balance set up in which the power in a given interval is given by the net of the power into the interval from other intervals minus the power transmitted to other intervals minus losses to (and/or any gains from) the outside. For the stationary state the above description implies that we can subdivide the total frequency range into intervals of constant power.

$$\int_{\Delta f_i} P(f)\, df = \text{const.} \qquad (3.1)$$

Such a relation can always be written provided a power spectral density exists. However the choice of the Δf_i will depend upon the physical processes involved in the noise. For the case in which the average current deviation is assumed

to have a nonlinear behavior, we want the different frequency ranges to be treated in a way that gives equal weight to each of them. This can be accomplished by assuming Eq. (3.1) applies for the intervals arising for constant frequency resolution.

This means that Δf_i is proportional to a characteristic frequency of the interval. It then follows from (3.1) by self-consistent iteration[16] that

$$P(f) \propto f^{-1} \tag{3.2}$$

More generally, the frequency intervals involved in Eq. (3.1) may not be those of constant frequency resolution. Thus small deviations from the exact 1/f functional dependence can be easily accounted for by slightly modifying the choice of frequency interval.

Alternatively, if the intervals are to remain simple the relation may not be so simple. Thus we might use a modulation function $T(f)$ under the integral sign to obtain

$$\int_{\Delta f_i} T(f) \quad P(f)\, df = \text{const.} \tag{3.3}$$

In this way, a wide range of spectral laws for $P(f)$ can occur depending on the choice of $T(f)$ even when Δf_i is contrained to be a constant frequency resolution interval.

An alternative way of viewing the 1/f problem is to start with Eq. (3.1) and impose a 1/f power spectral density. Then it follows that the systematically chosen spectral intervals of integration are just those which give constant frequency resolution.

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12. If K is even, the division of T_o into K parts includes one part made up of segments of length T/2K respectively at the beginning and the end of the T_o interval
13. Note by emphasizing the point t_a and nesting the intervals in a symmetrical fashion about it, we depart somewhat from the nesting procedure used by v. Weizsäcker. In v. Weizsäcker's case, the nesting is not symmetric but eventually closes down on a point. Since we are interested in the deviation at a point in time, we choose to build our nesting structure about that point.

14. A more precise statement is that I_n^a only has significant contributions in the frequency interval $\Delta f_n' = \alpha \Delta f_n$ where $\alpha > 1$ is independent of n. For simplicity in our development we take $\alpha = 1$.

15. If we had more realistically allowed I_n^a to provide a contribution [illegible] frequency interval $\Delta f_n' = \alpha \Delta f_n$, we would have [illegible] the [illegible] that makes up the average [illegible] deviation so that each individual term would contribute only to a single non-overlapping constant frequency resolution interval.

16. If we assume $P(f)$ is slowly varying over $\Delta f_i = (R - 1)f/R$, then a first iteration of Eq. (3.1) yields $P_1 = \text{const.}\ R/(R - 1)f$. Insertion of $P_1(f)$ in the integral yield $[R/(R - 1)][\ln(2R - 1)/R]$ const. Thus a final self-consistent form for $P(f)$ is const. $[\ln(2R - 1)/$[illegible]$]^{-1}/f$. Note a similar result is obtained if we choose the integration interval to be proportional to Δf_i with factor of proportionality independent of i.